RIGOR AND INTUITION IN TOPOLOGY: *THE DEVELOPMENT OF THE POINCARÉ CONJECTURE*

Abstract: In this work, I study and develop the concept of rigor in mathematics and its relation to intuition with a particular focus on Poincaré's topological work. The arithmetization project of the 19th century has become the paradigm of rigor in mathematics. The arithmetization project aimed to reduce analysis to arithmetic and was motivated by the discovery of examples that contradicted our intuitions about continuous functions. In order to remedy and avoid these contradictions, mathematicians became especially interested in introducing rigor to analysis, where rigor is taken to be something that sharpens and clarifies the definitions of our intuitive notions. So, on this picture we develop mathematical concepts by first considering our intuitive ideas and then we use rigor to make these ideas precise and insusceptible to counterexamples.

This exhibits one common way in which rigor is used in mathematics, but it is not the only way and it is not the paradigm in contemporary mathematics because it downplays the significance of intuition. Instead, we see something closer to the contemporary paradigm in Poincaré's development of his famous conjecture, which required a harmony of rigor and intuition. He develops this conjecture in his *Analysis Situs* and its five supplements, throughout which both rigor and intuition remain important. Poincaré does not disregard his intuition once he introduces rigorous definitions but rather he continuously considers his intuitions. For Poincaré, rigor is not a remedy for the failures of intuition but rather works together with intuition in order to achieve successful mathematical development.

Thus, we see the importance of a harmony of rigor and intuition in Poincaré's work, an importance which applies more generally. Throughout the 20th century, contemporary mathematics has become increasingly abstract. We see in the Poincaré example that given the abstract nature of algebraic topology it was necessary to first develop the subject matter —— i.e., the concepts of homotopy and homology —— and this is where the harmony of rigor and intuition plays an important role. This need to develop the subject matter of abstract mathematics has remained present in contemporary mathematics and so the harmonious interplay of rigor and intuition has remained important.