THE EXPLANATORY VALUE OF CATEGORY THEORY

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ABSTRACT. Category theory has proven to be applicable across all of mathematics. In some sense this is not surprising because category theory was created for the purpose of application (specifically, application to algebraic topology). But I will argue that the significance of category theory extends past its applicability — in particular, there is a significant *explanatory* benefit. The question of what constitutes a mathematical explanation is of perennial interest to philosophers. Reflection on category theory's unique role in mathematics unearths some features of mathematical explanation that are not often made explicit and that philosophers have tended not to notice.

There are many ways that category theoretic methods provide explanations. For instance, important results in different areas of mathematics are unified by the fact that they are all corollaries of the same category theoretic theorem, such as the theorem that right adjoints preserve limits. Or consider the ways of defining structures in category theory with universal properties — the whole perspective sheds light on how constructions from different domains are related to one another. The categorical product for instance, unites many seemingly unrelated mathematical constructions such as the Cartesian product, union, and conjunction. Such examples introduce both generalization and unification within mathematics. Moreover, this unification allows for meaningful and surprising mathematical analogies to arise. These generalizations and analogies are explanatory and result from the structural features of category theory.

In order to highlight the explanatory value of category theory, I will first provide a characterization of the structure unique to category theory. It is this structure that makes category theory apt for producing explanations. With a clear picture of category theoretic structure, I will present a few examples that illustrate how category theory proves to be explanatory — in particular, how the structural features of category theory are explanatory.